EVAPORATION CONSTANTS FOR DROPS OF WATER AND ETHYL ALCOHOL IN NATURAL GAS COMBUSTION PRODUCTS AND IN HEATED AIR

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A formula is obtained for computing the evaporation constants for water and ethyl alcohol in high-temperature gaseous media. The computed evaporation constants are compared with experimental values.

A special feature of problems involving complex heat and mass transfer, occurring in evaporation of drops of pure substances or solutions in moving high-temperature radiating media (in the combustion products of various types of fuels, in heated air, or other gases), is well known to be the fact that the systems of equations describing the heat and mass transfer process in these media involve nonlinear integro-differential equations, for which there are no general methods of solution at present. This kind of problem is usually solved in a simplified formulation, and the following assumptions are very often made: a one-dimensional model is taken for the transfer of heat by conduction and radiation, and the resulting heat and mass transfer is evaluated from relations which are convenient to apply.

These assumptions are made in the present work in examining heat transfer of a drop of liquid evaporating in a high-temperature stream. The total heat flux to the drop surface is assumed to consist of three independent fluxes: conductive, convective, and radiative. By assuming that the heat fluxes are independent one can determine them by theoretical methods, compare them with experimental measurements, and thus subsequently represent the results of calculation of a complex heat and mass transfer problem in a form convenient for engineering application. We note that because of the extremely complex mechanism involved in evaporation of a drop in moving high-temperature media, no rigorous theory describing the kinetics of the evaporation process has presently been developed.

For an evaporating spherical drop in a steady high-temperature gas stream the following equations are valid:

a) for mass transfer by thermal motion

$$\frac{dm}{d\tau} \Delta H_v = 4\pi r_1^2 \lambda \left(\frac{dT}{dr} \right)_{r=r_1} + R_a, \tag{1}$$

where r_1 is the drop radius; ΔH_v is the heat of evaporation; R_a is the fraction of radiative heat from the gaseous medium and the hot walls of the combustion chamber; and $dm/d\tau$ is the rate of evaporation;

b) to determine temperature around the surface of the evaporating drop

$$r^2 \frac{d^2 T}{dr^2} + \left(2r - \frac{dm}{d\tau} \cdot \frac{C_p}{4\pi\lambda} \right) \frac{dT}{dr} = 0, \tag{2}$$

where C_p and λ are the heat capacity and the thermal conductivity in the boundary layer of thickness $r_2 - r_1$, formed during quasi-steady evaporation; and r_2 is the outer boundary of the boundary layer. The second term in the circular brackets in Eq. (2) takes account of the effect of rate of mass evaporation of the substance on the temperature distribution in the layer around the drop.

With the boundary conditions $r = r_1$, $T = T_n$ and $r = r_2$, $T = T_\infty$, the solution of Eq. (2) takes the form

$$\frac{T - T_n}{T_\infty - T_n} = \frac{\exp(-E/r) - \exp(-E/r_1)}{\exp(-E/r_2) - \exp(-E/r_1)}, \tag{3}$$

where $E = (dm/d\tau)/(C_p/4\pi\lambda)$.

Differentiating Eq. (3), we obtain an expression for the derivative dT/dr in the boundary layer around the drop:

$$\frac{dT}{dr} = \frac{\Delta T \exp(-E/r)}{\exp(-E/r_2) - \exp(-E/r_1)} \cdot \frac{E}{r^2}, \tag{4}$$

where $\Delta T = T_\infty - T_n$, which for $r = r_1$ determines the temperature gradient at the surface of the evaporating drop:

$$\left.\frac{dT}{dr}\right|_{r=r_1} = \frac{\Delta T}{\exp E \left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right) - 1} \cdot \frac{E}{r_1^2}. \tag{5}$$

Substituting Eq. (5) into Eq. (1), and performing certain transformations, we obtain an equation for the rate of evaporation of the drop

$$\frac{dm}{d\tau} = \frac{4\pi\lambda}{C_p \left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)} \ln\left(1 + \frac{C_p \Delta T}{\Delta H_v - a^*}\right). \tag{6}$$

Here $a^* = R_a/(dm/d\tau)$.

For steady quasi-stationary heat transfer in a high-temperature medium the heat supplied to the drop is dissipated completely in evaporation, i.e.,

$$Q = 4\pi r_1^2 \alpha \Delta T = \Delta H_v (dm/d\tau). \tag{7}$$

The dimensionless heat transfer coefficient for the evaporating drop, taking account of Eqs. (6) and (7), takes the form

$$Nu_1 = \frac{\alpha 2 r_1}{\lambda} = \frac{2\Delta H_v}{C_p \left(1 - \dfrac{r_1}{r_2}\right)\Delta T} \ln\left(1 + \frac{C_p \Delta T}{\Delta H_v - a^*}\right). \tag{8}$$

Since the heat transfer equation (8) for the evaporating drop included the effect of radiated heat and the rate of evaporation of mass from the drop surface on the heat transfer process, it can describe a wider circle of observed evaporation phenomena than the well-known evaporation equations [1-5, 8].

The effect of forced convection on heat transfer is accounted for in a number of experimental papers [5-8] by a factor f, a function of dimensionless heat transfer variables (Re and Pr):

$$f = 1 + 0.276 \, \mathrm{Re}^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{3}}. \tag{9}$$

Equation (9) was verified by comparison with experimental data of [7, 9, 10] on rate of evaporation of drops of water and ethyl alcohol in heated air.

With the correction f included, Eq. (8) takes the form

$$Nu = Nu_1 f. \tag{10}$$

Many experimental papers have given the following relationship between the rate of evaporation $dm/d\tau$ and the evaporation constant K_0:

$$dm/d\tau = K_0 (\pi\rho_1 r_1/2), \tag{11}$$

where ρ_1 is the density of the liquid at the surface temperature. An equation for the evaporation constants of varous substances in high-temperature gases can be obtained by simultaneous solution of Eqs. (5) and (11):

$$K_0 = \frac{8\lambda}{C_p \rho_1 \left(1 - \dfrac{r_1}{r_2}\right)} \ln\left(1 + \frac{C_p \Delta T}{\Delta H_v - a^*}\right). \tag{12}$$
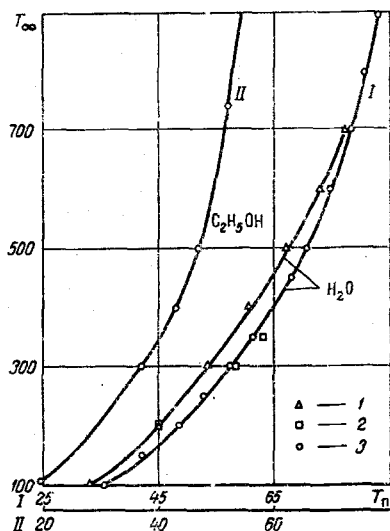
Fig. 1

Fig. 2

Fig. 1. Experimental dependence of the surface temperature of evapora-
ting drops of water and ethyl alcohol on the temperature of the surround-
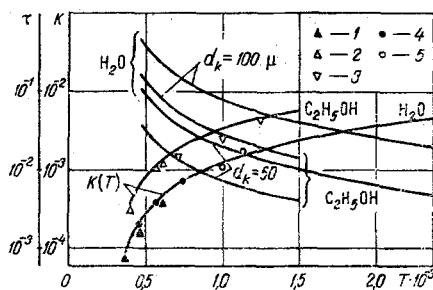ing medium: 1) [1]; 2) [7]; 3) [9]. T is expressed in °C.

Fig. 2. Calculated and experimental values of the evaporation constants
for drops of water and ethyl alcohol in heated air: 1) experimental data
of [1]; 2) of [7]; 3,4) of [9]; 5) of [10]. (The evaporation constants were
obtained from experimental data on the rate of evaporation of drops of
diameter 1 mm). The curves were calculated from Eq. (16). K, cm^2
/sec; τ, sec; $T \cdot 10^{-3}$, °K; d_c, μ.

Allowing for the correction factor, the evaporation constant for a substance in a moving medium can be
written as:

$$K_v = K_0 f, \tag{13}$$

where K_0 is the evaporation constant of the substance in a medium at rest, as calculated from Eq. (12).
The time for a drop of initial size d_0 to evaporate to an ambient size d_1 can be determined from the Srez-
nevskyi Law, which has been confirmed in numerous experiments on evaporation and combustion [1, 9, 12,
13]:

$$\tau \text{ evap} = (d_0^2 - d_1^2)/K_0. \tag{14}$$

According to Eq. (14), the time for complete evaporation (or the "lifetime") for a fully evaporated drop
(with $d_1 = 0$) is

$$\tau \text{evap} = d_0^2/K_0. \tag{15}$$

Assuming that the Sreznevskyi law is valid for a drop evaporating in a moving medium (within a nar-
row range of the speed of the moving medium) the time for the drop size to change and the lifetime are de-
termined from Eqs. (14) and (15), using the relationship between K_V and K_0 from Eq. (13).

References [1, 7, and 9] contain the most complete results of an experimental study of evaporation
of drops of water and ethyl alcohol. The experimental values in these papers are shown in Fig. 1 and 2,
along with our computations performed according to the above equations. It can be seen from Fig. 2 that
the evaporation constants for water in heated air obtained in [7] and [9] differ somewhat. In the tempera-
ture range compared for air the data of [1*] were considerably above those of [7, 9]. In the author's opin-
ion, the experimental results of [9], obtained under steady conditions of evaporation, seem the more
reliable.

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*The evaporation constants in [1] for a medium at rest were determined for every isotherm by extrapolating
the relationship $K_V = f(v)$ (where v is the air velocity) to a zero value of the air stream velocity. The
evaporation constants for these conditions (v = 0) given in [10] were calculated using a correction factor f,
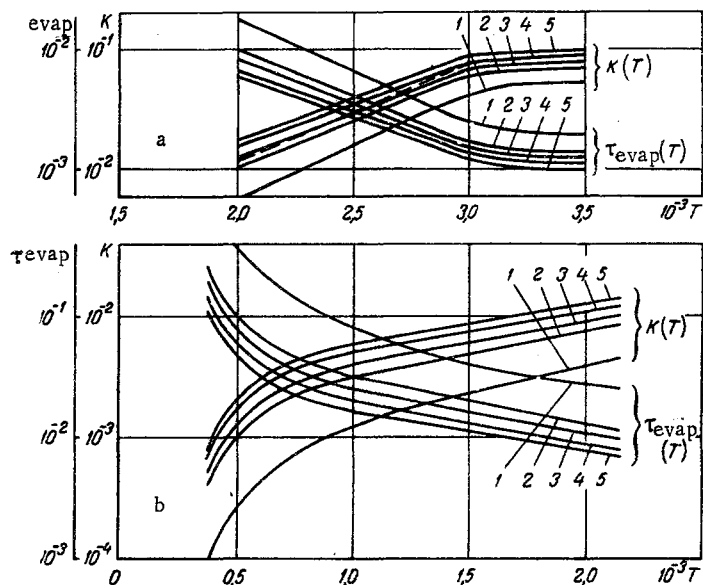computed according to Eq. (9).

Fig. 3. Evaporation constants and times for complete evaporation of water drops of initial diameter 100 μ in combustion products of natural gas in air (a), and in heated air (b): 1) $V_{rel} = 0$ m/sec; 2) 50; 3) 100; 4) 200; 5) 300. The broken line indicates alcohol with $V_{rel} = 0$. K, cm^2/sec; τ, sec; $T \cdot 10^{-3}$, °K.

Experimental data on evaporation constants for water in high-temperature flow at temperatures exceeding 1000°C are the only ones available, although the need for such data remains very great, because high-temperature vapor mixtures are generated and used in a number of processes [4].

Many tests on the rate of evaporation of drops of pure substance and of different kinds of liquid fuels under steady evaporation conditions [12,14, et al.] show that the evaporation process for a spherical drop takes place in the spherical layer formed around the drop, whose outer radius r_2 is equal to three drop radii, independent of variation of the drop size during the entire evaporation time, i.e., $r_2 = 3r_1$ (where r_1 is the drop radius). This experimentally established relation between the size of an evaporating drop and the boundary layer surrounding it was included in Eq. (12), and resulted in an equation for calculating the evaporation constants of drops of different substances:

$$K_0 = \frac{12\lambda}{C_p \rho_1} \ln \left(1 + \frac{C_p \Delta T}{\Delta H_v - a^*} \right).$$
(16)

The reliability of calculating evaporation constants of substances using Eq. (16) was checked by comparing evaporation constants evaluated from this formula for water and ethyl alcohol with experimental values of these quantities in [7,9]. In calculating the evaporation constants for water and ethyl alcohol using Eq. (16) in heated air an experimental dependence was used for the drop surface temperature on the medium temperature [9], and the quantity a^* was neglected. We will show below that this assumption is valid for evaporation in air. The results of the comparison are shown in Fig. 2, from which it follows that the calculated evaporation constants and the experimental values [9] for evaporation of water and ethyl alcohol show good agreement.

Figure 2 also shows the time for complete evaporation of drops of water and ethyl alcohol of initial diameters of 50 and 100 μ in air heated to 2400°K at atmospheric pressure. As the temperature of the medium increases the time for complete evaporation of the drop rapidly reduces. For water drops of initial diameter 100 μ, as the medium temperature increased from 500 to 2400°K, the evaporation time reduced by a factor of 20, and for drops of water of the same diameter, the evaporation time was reduced by a factor of 9 when the temperature changed from 500 to 1500°K.

In the same medium evaporation of a drop of alcohol takes place more than 2.5 times faster than for a drop of water of the same size. This is due to the lower heat of evaporation of ethyl alcohol in comparison with that of water.

TABLE 1. Calculated Emissivities for Combustion
Products

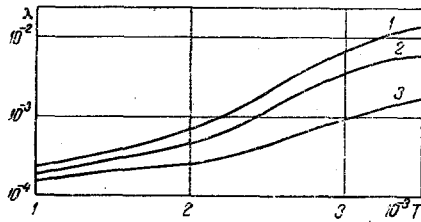| Emissivity	Equilibrium temperature of combustion products, °K		
	2000	2500	3000
	Methane in air		
ε_{red}	0,27	0,18	0,10
	Kerosene in oxygen		
ε_{red}	0,48	0,38	0,30

Fig. 4. Thermal conductivities for combustion products of natural gas in oxygen and heated air: 1) natural gas + air; 2) natural gas + O_2; 3) dry heated air. λ, cal/cm·sec ·deg; $T \cdot 10^{-3}$, °K.

Figure 3a, b show the evaporation constants and the lifetimes of water drops of initial diameter 100 μ in the combustion products of methane and air and in heated air, for relative flow rates over the drops of 50, 100, 200, and 300 m/sec. The correction factor f was computed using Eq. (9), the lifetime using Eq. (15), and the evaporation constants using Eqs. (12) and (13).

The evaporation constants for water in the combustion products of methane in air are higher than the corresponding values for evaporation in heated air, which is due to the higher heat conduction in the combustion products.

The thermal conductivities of combustion products of natural gas in air and oxygen and heated air are shown in Fig. 4. By comparing the thermal conductivities of combustion products one can draw the qualitative conclusion beforehand that the evaporation constants for air in the combustion products of methane in air will be higher than in the combustion products of methane in oxygen. The thermal conductivities of the combustion products of methane in air and in oxygen were taken from [15], and for air from [16].

The value of the parameter $a*$, which accounts for the fraction of radiant energy in the total heat reaching the surface of the evaporating drop, was calculated for drops of water and ethyl alcohol of diameter 100 μ, evaporating in the combustion products of methane in air and of kerosene in pure oxygen, at the equilibrium combustion temperature of 2000, 2500, and 3000°K. The radiant flux R_u to the drop surface from the combustion products is given, to a first approximation, by the Stefan—Boltzmann law. The emissivities given for the system consisting of the drop surface F_k surrounded by a spherical shell of combustion products of layer thickness l, were calculated in the usual way, as recommended in [17]. The main radiating combustion products at the above temperatures were assumed to be the molecules of CO_2, H_2O, and CO. The calculated values of emissivity for a drop surrounded by a spherical shell of combustion products of methane in air and of kerosene in pure oxygen are shown in Table 1. The data on the composition of combustion products of methane in air and in oxygen, required to calculate the emissivities, were taken from [16], and for heated air, from [18]. The thickness of the radiating layer of combustion products was taken to be 10 cm, since the emissivities of the CO_2, H_2O, and CO molecules are known for this thickness. Since the calculations made on the effect of radiant heat flux from the combustion products on the heat transfer of the evaporating drop are qualitative, simple equations of radiant transfer were used for the calculation.

The effects of radiant heat flux and rate of evaporation on the evaporation process were not taken into account in [1-3, 6, 7, et al.].

The evaporation constants and lifetimes for water drops in combustion products of natural gas and in heated air at atmospheric pressure over a wide temperature range (up to 2400°K) are quite reliable and are important [4].

Calculations made have shown that the value of $a*$ does not exceed 8% of the heat of evaporation ΔH_V in the case of evaporation of drops of water in combustion products of methane in air, in the temperature range 2000-3000°K, and does not exceed 14% for evaporation in the products of combustion of kerosene in pure oxygen over the same temperature range. In the case of evaporation of drops of alcohol of the same size in the combustion products of methane in air the ratio $a*/\Delta H_V$ increases to 23%.

Since the parameter $a*$ is in the numerator of the logarithmic expression in Eq. (12), the evaporation constant will be larger, the smaller the heat of evaporation of the drop substance, for constant value of $a.*$

A decrease in the difference of the quantities $(\Delta H_V - a*)$ by 8% leads to an increase in the constant of evaporation of water in combustion products of methane in air at $T = 2000°K$ of approximately 4%, and an increase in the constant of evaporation of alcohol† at the same temperature of approximately 6%. In evaporating water and alcohol in the combustion products of kerosene in pure oxygen, for which the concentration of molecules of CO_2, H_2O, and CO is considerably greater than in the combustion products of methane in air and in oxygen, one would expect an increase in the evaporation constant of 8-10% for alcohol drops of diameter 100μ. The slower rate of increase of the evaporation constant for water is due to the higher heat of evaporation in comparison with alcohol. This comparison enables us to conclude that the use of water as a solvent for the alkali metals, followed by injection of water solutions of these salts into the combustion products of various fuels, with the objective of increasing their electrical conductivity, is not the best procedure.

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†The heat of chemical reaction of combustion was neglected in computing the evaporation constant for alcohol.